חAmIBIA UחIVERSITY OF SCIEПCE AחD TECHחOLOGY

## FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

## SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF BIOLOGY, CHEMISTRY AND PHYSICS

| QUALIFICATION: BACHELOR OF SCIENCE |  |
| :--- | :--- |
| QUALIFICATION CODE: 07BOSC | LEVEL: 7 |
| COURSE NAME: QUANTUM CHEMISTRY AND <br> MOLECULAR SPECTROSCOPY | COURSE CODE: QCM701S |
| SESSION: JUNE 2023 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER |  |
| :--- | :--- |
| EXAMINER(S) | Prof Habauka M Kwaambwa |
| MODERATOR: | Prof Edet F Archibong |

## INSTRUCTIONS

1. Answer ALL the SIX questions
2. Write clearly and neatly
3. Number the answers clearly
4. All written work must be done in blue or black ink
5. No books, notes and other additional aids are allowed
6. Mark all answers clearly with their respective question numbers

## PERMISSIBLE MATERIALS

Non-programmable Calculators
ATTACHMENT
List of Useful Constants
THIS QUESTION PAPER CONSISTS OF 7 PAGES (Including this front page and List of Useful Constants)
(a) Define the term blackbody radiation.
(b) Explain how classical mechanics failed to explain the phenomenon of blackbody radiation and how this phenomenon contributed to the development of quantum mechanics. (3)
(c) The Rayleigh-Jeans law of a blackbody radiation as function of wavelength is given as:

$$
\mathrm{U}(\lambda)=\frac{8 \pi \mathrm{kT}}{\lambda^{3}}
$$

Deduce the corresponding expression and SI units of the energy density, $\mathrm{U}(v)$, as function of frequency, $v$.
(d) Under what condition does the Rayleigh-Jeans law of a blackbody radiation as function of frequency agree with blackbody radiation experimental results.
(e) Sodium metal with a work function of 2.28 eV gives off photoelectrons when it is bombarded with ultraviolet radiation of 200 nm .
(i) Calculate the maximum possible kinetic energy and velocity of photoelectrons emitted from Na exposed to 200 nm of light.
(ii) Calculate the wavelength required to cause photoelectron emission from Na .
(f) The derivation by Bohr of the hydrogen atom given below.
$\bar{v}=R_{g}\left(\frac{1}{n_{1}^{2}}-\frac{1}{\mathrm{n}_{2}^{2}}\right)$, where $\mathrm{R}_{\mathrm{g}}=109677.58 \mathrm{~cm}^{-1}$
Calculate the wavelength, $\lambda$ (in $\mathbf{n m}$ ) and ionisation energy (in $\mathbf{e V}$ ) for the Lyman line of the Hemission. (The $n_{1}=2$ for the Balmer series).

The free-electron molecular orbital (FEMO) method, which assumes that the $\pi$ electrons are trapped in a 1-D box of length assumed to be $9.8 \AA$ and if the $\lambda_{\text {max }}$ was measured to be 352 nm , answer the following questions:
(a) Calculate the zero-point energy (in eV ) of the system.
(b) Why is the zero-point energy equal to zero not feasible?
(c) Determine the transition for the wavelength $\left(\lambda_{\max }=352 \mathrm{~nm}\right)$ measured.
(6)
(d) Determine the number of $\pi$ electrons and identify the HOMO and LUMO.
(3)
(e) What is the main source of error of the model in predicating the maximum wavelength of absorption of conjugated diene molecules?
(f) For 3-dimension cubic particle-in-a-box of length $9.8 \AA$, calculate the ground state energy (in eV )?
(g) From the above information and calculations, the most likely conjugated diene(s) to fit the model is/are:
(i) $\mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}_{2}$
(ii) $\left(\mathrm{CH}_{3}\right)_{2} \mathrm{~N}^{+}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\mathrm{N}\left(\mathrm{CH}_{3}\right)_{2}$
(iii) $\mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}_{2}$
(a) Is the wave function shown in the diagram well-behaved or not well-bahaved? State the reason(s) for your answer.

(b) The wave function, $\Psi$, for an electron in the highest occupied molecular orbital of polydiene based on 1-dimensional particle-in-a-box model is given by:

$$
\Psi(x)=\left(\frac{2}{L}\right)^{\frac{1}{2}} \sin \left(\frac{5 \pi x}{L}\right) \text { for } 0 \leq x \leq 40 n m
$$

(i) Plot the variation of $\Psi(\mathrm{x})$ and $\Psi^{2}(\mathrm{x})$ for a particle-in-a-box for

$$
\begin{equation*}
0 \leq x \leq 40 \mathrm{~nm} \tag{3}
\end{equation*}
$$

(ii) At what values of $\mathbf{x}$ is $\Psi(\mathrm{x})$ equal to zero, minimum and maximum, and the probability of finding the particle (probability density function) maximum in the range $0<x<40 \mathrm{~nm}$ ?
(iii) What is the probability of finding the particle in the range $L / 10 \leq x \leq L / 2$ ?
(c) State using a mathematical expression what is meant in quantum theory for each of the following:
(i) Operator $\hat{A}$ is linear to the wave functions $\Psi_{i}$ and $\Psi_{j}$.
(ii) Wave functions $\Psi_{1}$ and $\Psi_{2}$ are orthogonal.
(iii) Operators A and B commute of wave function $\Psi$.
(iv) Hermitian operator $\hat{A}$ of wave functions $\Psi_{i}$ and $\Psi_{j}$.
(v) Expectation value, $\langle\mathrm{a}\rangle$, of the observable $A$ derived from a normalised wave function $\Psi$.
(d) What are the physical meanings of commuting operators and orthogonal wave functions in Quantum mechanics?
(e) Give the Schrödinger equation for a plane rigid rotor (circular motion in a fixed plane) and show that $\Psi=\frac{1}{\sqrt{2 \pi}} e^{i m_{1} \phi}$ is an acceptable solution of the of the operator form of the plane rigid rotor Schrödinger equation. What is the eigenvalue expression?

## QUESTION 5

(a) Which of the species would possess
(i) Rotational microwave spectrum?
(ii) Vibrational (infrared) spectrum?
$\mathrm{N}_{2}, \mathrm{IBr}, \mathrm{CO}_{2}$,
(b) Define the term selection rule using the following as examples:
(i) Particle-in-box
(ii) Plane rigid rotor
(iii) Simple harmonic oscillator
(iv) Anharmonic oscillator
(v) Rigid rotor - Harmonic Oscillator
(c) If the degeneracy in pure rotational energy states is 7, what is the value of the quantum number $J$ and the possible values of the quantum number $m_{1}$ ?
(d) The ro-vibrational spectrum is divided into three branches, namely, $P, Q$ and $R$. What is the approximate separation in terms $B$ between the innermost line of the $P$ and second innermost line of the $R$ branch?
(e) From the ro-vibrational spectrum of ${ }^{1} \mathrm{H}^{127}$, the spacing between the peaks was approximated to be $13.1 \mathrm{~cm}^{-1}$. Answer the following questions:
(i) Deduce the moment inertia, I , of ${ }^{1} \mathrm{H}^{127}$ I.
(ii) Calculate the reduced mass of ${ }^{1} \mathrm{H}^{127}$.
(iii) Evaluate the internuclear distance (in $\AA$ ) of ${ }^{1} \mathrm{H}^{127}$ I. Atomic masses (amu): $\quad{ }^{1} \mathrm{H}=1.0079 \quad{ }^{127} \mathrm{I}=126.90447$
(f) The force constant of ${ }^{79} \mathrm{Br}^{79} \mathrm{Br}$ is $240 \mathrm{Nm}^{-1}$. Calculate the fundamental vibrational wavenumber of $\mathrm{Br}_{2}$. Given: RAM for $\mathrm{Br}=78.92 \mathrm{amu}$

## END OF EXAM QUESTIONS

## LIST OF USEFUL CONSTANTS:

| Universal Gas constant | R | $=8.314 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ |
| :--- | :--- | :--- |
| Boltzmann's constant, | k | $=1.381 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$ |
| Planck's constant | h | $=6.626 \times 10^{-34} \mathrm{~J} \mathrm{~s}$ |
| Debye-Huckel's constant, | A | $=0.509\left(\mathrm{~mol} \mathrm{dm}^{-3}\right)^{1 / 2}$ or mol${ }^{-0.5} \mathrm{~kg}^{0.5}$ |
| Faraday's constant | F | $=96485 \mathrm{C} \mathrm{mol}^{-1}$ |
| Mass of electron | $\mathrm{m}_{\mathrm{e}}$ | $=9.109 \times 10^{-31} \mathrm{~kg}^{2}$ |
| Velocity of light | c | $=2.998 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ |
| Avogadro's constant | $\mathrm{N}_{\mathrm{A}}$ | $=6.022 \times 10^{23}$ |
| 1 electron volt $(\mathrm{eV})$ |  | $=1.602 \times 10^{-19} \mathrm{~J}$ |

